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Eingang:
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Mainz Microtron MAMI

Collaboration A2: “Real Photon Experiments”
Spokesperson: R. Beck

Proposal for an Experiment

Test of Chiral Perturbation Theory and C and CP Invariance in Eta Meson Decay

Collaborators :

CrystalBall@MAMI collaboration

Spokespersons for the Experiment : B. Nefkens - Los Angeles, R. Beck - Mainz

Abstract of Physics :

We propose to investigate six neutral decay modes of the eta meson; all are measured simultaneously. This experiment has two phases. In Phase I, to be performed with MAMI-B in 2004, our priority is the measurement of the branching ratio and decay spectrum of $\eta \to \pi^0\gamma\gamma$. The decay amplitude is determined by the third-order term in the momentum expansion; the first term is zero and the second is small. Thus $\eta \to \pi^0\gamma\gamma$ is a unique test of Chiral Perturbation Theory. We will also obtain a new value for the slope parameter in $\eta \to 3\pi^0$, which is another test of ChPT. Etas are photoproduced in the reaction $\gamma p \to \eta p$ with tagged photon of 720 to 820 MeV. The η decays and recoil proton are measured with a multi-element detector, consisting of Crystal Ball, TAPS, DAPHNE tracker and a scintillator PID. In Phase II, we want to improve by a factor 10-20 three tests of charge conjugation invariance, namely $\eta \not\to 2\pi^0\gamma$, $\eta \not\to 3\pi^0\gamma$, and $\eta \not\to 3\gamma$, and make a unique test of CP invariance, namely $\eta \not\to 4\pi^0$. We will accumulate as many etas as possible, running at least 1000 hours when MAMI-C is installed.

Abstract of Equipment :

We require a beam of tagged photons incident on a liquid-hydrogen target and the detection 4π Crystal Ball photon spectrometer in combination with TAPS as forward wall, the DAPHNE tracker and a scintillator PID. The Glasgow tagging system will provide the intense photon beam.

MAMI–Specifications :

beam energy	880 MeV
beam current	< 100 nA
time structure	cw
polarization	

Experiment–Specifications :

experimental hall/beam	A2
detector	Crystal Ball and TAPS as forward wall
target material	liquid hydrogen

Beam Time Request :

set–up without beam	will be done in conjunction with the CB/TAPS installation
set–up/tests with beam	30 hours
data taking	240 hours (Phase I) 1000 hours (Phase II)

November 24, 2003

Mainz Microtron MAMI

Title: **Test of Chiral Perturbation Theory and C and CP Invariance in Eta Meson Decay**

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Time Request: Phase I: **270** hours with **MAMI-B** in 2004
(200 h data, 40 h empty target, 30 h setup)
Phase II: **1000** hours with **MAMI-C**

Abstract

We propose to investigate six neutral decay modes of the eta meson; all are measured simultaneously. This experiment has two phases. In Phase I, to be performed with MAMI-B in 2004, our priority is the measurement of the branching ratio and decay spectrum of $\eta \to \pi^0\gamma\gamma$. The decay amplitude is determined by the third-order term in the momentum expansion; the first term is zero and the second is small. Thus, $\eta \to \pi^0\gamma\gamma$ is a unique, sensitive test of Chiral Perturbation Theory (χPT). We will also obtain a new value for the slope parameter in $\eta \to 3\pi^0$, which is another test of χPT. Etas are photoproduced in the reaction $\gamma p \to \eta p$ with tagged photons of 720 to 820 MeV. The η decays and recoil proton are measured with a multi-element detector, consisting of the Crystal Ball, TAPS, DAPHNE tracker and a scintillator PID all of which are being assembled in the A2 hall.

In Phase II, we want to improve by a factor of $10-20$ three tests of charge conjugation invariance, namely $\eta \not\to 2\pi^0\gamma$, $\eta \not\to 3\pi^0\gamma$, and $\eta \not\to 3\gamma$, and make a unique test of CP invariance, namely $\eta \not\to 4\pi^0$. We will accumulate as many etas as possible, running at least 1000 hours when MAMI-C is installed.

1 Introduction

The eta is a unique meson because it provides a very sensitive test of chiral perturbation theory, χPT. The decay rate and the Dalitz plot of $\eta \to \pi^0\gamma\gamma$ are determined by the third order term of χPT. Another test of χPT is provided by the slope parameter of $\eta \to 3\pi^0$. Selected eta decays such as $\eta \to 3\gamma$, $\eta \to 2\pi^0\gamma$, and $\eta \to 3\pi^0\gamma$ are forbidden to occur by charge conjugation, invariance of the flavor conserving electrostrong interaction. A sensitive search for those forbidden η decays gives important new limits on C-invariance. Finally, the decay mode $\eta \to 4\pi^0$ is forbidden by CP-invariance.

In the CB experiment of the AGS the above six eta decays together with several others were successfully studied. They were investigated simultaneously which resulted in a substantial reduction in running and analyzing time.

Below we discuss each of the six decays separately and show why a new investigation of $\eta \to \pi^0\gamma\gamma$ is urgently needed and better data on the other decay modes are highly desirable.

71.6% of all η decays result in neutral particles — photons and π^0's. The neutral decay modes are listed in Table 1, which also shows the physics theories and symmetries which can be investigated for each mode.

1.1 $\eta \to \pi^0\gamma\gamma$ decay

One of the important decay modes of the eta meson is the doubly radiative decay,

$$\eta \to \pi^0\gamma\gamma,$$

because it provides us with a "gold-plated" test of Chiral Perturbation Theory, χPT. Basic chiral symmetry does not allow an electromagnetic contribution to the η decay amplitude in leading order in the momentum expansion [1, 2]. The next order is much suppressed because it is G-parity violating. Thus the third order, $\mathcal{O}(p^6)$, makes the major contribution to $\eta \to \pi^0\gamma\gamma$ decay [3]. Until now the calculations of some of the third-order terms are somewhat model-dependent and their correctness needs experimental verification. Since χPT is the leading theory for low energy QCD calculations, its validity must be checked up to the third order.

Two rare eta decays, $\eta \to \pi^0e^+e^-$ and $\eta \to \pi^0\mu^+\mu^-$, provide a new sensitive probe of charge conjugation, or C-invariance of the electromagnetic interaction of hadrons [4]. Calculation of the sensitivity of these tests are based on the amplitude for $\eta \to \pi^0\gamma\gamma$. The related rare decay $K_L^0 \to \pi^0e^+e^-$ provides a test of direct CP violation [5] and again good data on $\eta \to \pi^0\gamma\gamma$ are needed to properly assess the sensitivity of this test.

Table 1: The Neutral η Decays.

Decay Mode	Branching Ratio	Physics highlight
All Neutrals	$(71.6 \pm 0.4)\%$	
2γ	$(39.3 \pm 0.3)\%$	$SU(3)$ octet-singlet mixing
$3\pi^0$	$(32.2 \pm 0.3)\%$	χPTh; $m_u - m_d$
$\pi^0\gamma\gamma$	$(3.2 \pm 0.9) \times 10^{-4}$	$\chi PTh,\ O(p^6)$
$2\pi^0$	$< 4.3 \times 10^{-4}$	P and CP invariance
$4\pi^0$	$< 6.9 \times 10^{-7}$	P and CP invariance
$\pi^0\pi^0\gamma$	$< 5 \times 10^{-4}$	C (isoscalar) invariance
$\pi^0\pi^0\pi^0\gamma$	$< 4.7 \times 10^{-5}$	C (isovector) invariance
3γ	$< 4.5 \times 10^{-5}$	C (isovector, isoscalar)
4γ	$< 2.8\%$	
$\pi^0\pi^0\gamma\gamma$	$< 3.1 \times 10^{-3}$	χPTh, New Physics
$\nu_e\bar{\nu}_e$	$< 2.8\%$	New Physics (leptoquarks)
$\nu_e\bar{\nu}_\mu$	$< 2.8\%$	New Physics (leptoquarks)
$\nu_e\nu_e$	$< 2.8\%$	New Physics (leptoquarks)
$\gamma\nu\nu$	$< 2.8\%$	New Physics (leptoquarks)
$\pi^0\nu\bar{\nu}$	$< 2.8\%$	New Physics (leptoquarks)

A summary of the important published theoretical predictions for the decay rate of $\eta \to \pi^0\gamma\gamma$ are given in Table 2. They involve sophisticated χPT calculations, also the versatile vector-meson-dominance model results and an upper limit based on Q-box calculations. An extensive discussion of the many calculations and the early experimental efforts to measure the rate is given in Ref. [6]. There are also new calculations underway. The general consensus is that the expected decay rate is $\Gamma(\eta \to \pi^0\gamma\gamma) = 0.42 \pm 0.20$ eV where the error has been purposely somewhat increased to encompass the GAMS results.

The Review of Particle Physics of 2002 [7] lists only one successful measurement of the branching ratio for $\eta \to \pi^0\gamma\gamma$. It was performed by the GAMS 2000 Collaboration at Serpukov. 38 events were found, and after much analysis [8] and reanalysis [9] the result is $\Gamma_{ex}(\eta \to \pi^0\gamma\gamma) = 0.84 \pm 0.14$ eV. This is twice the value calculated in the best theoretical effort, the third order χPT prediction. The Crystal Ball Collaboration at the AGS (CB@AGS), using a sample of about 19×10^6 etas, has detected some 500 $\eta \to \pi^0\gamma\gamma$ events, resulting in a branching ratio, BR, which is $BR(\eta \to \pi^0\gamma\gamma) = (3.2 \pm 0.9) \times 10^{-4}$, which with $\Gamma(\eta \to all) = 1.29 \pm 0.07\,\mathrm{keV}$ yields $\Gamma(\eta \to \pi^0\gamma\gamma) = 0.41 \pm 0.12$ eV [10, 11]. This is in excellent agreement with the χPT calculations. The larger result from GAMS 2000 is believed to have a serious background from $\eta \to 3\pi^0$ decays with overlapping clusters. Other efforts have been unsuccessful, e.g. the SMD detector at VEPP-2M has reported only an upper limit, $BR(\eta \to \pi^0\gamma\gamma) < 8.4 \times 10^{-4}$ [6].

A new experiment is needed to measure the decay spectrum as well as the branching ratio. The spectral shape is of course needed for a good determination of the branching ratio. Incidentally, the spectral shape of $\eta \to \pi^0\gamma\gamma$ is given by another third-order χPT calculation as illustrated in Fig. 1 and thus provides a somethat test of χPT.

1.2 The slope of $\eta \to 3\pi^0$, another test of χPT

An interesting parameter to investigate is the slope of the Dalitz plot for $\eta \to 3\pi^0$. In lowest order the Dalitz plot should be uniform because of the three identical π^0's in the final state. However, the $\pi - \pi$ interaction is strong and heavily energy dependent, which results in a tiny nonuniformity of the Dalitz plot. To incorporate the fact that there are 3 identical particles in

Table 2: Theoretical Predictions for the $\eta \to \pi^0\gamma\gamma$ decay rate.

Theory	$\Gamma(\eta \to \pi^0\gamma\gamma)$ (eV)	Reference
χPT, $\mathcal{O}(p^2)$	0	Y. Nemoto, Phys. Rev. D **54**, 6777 (1996)
χPT, $O(p^4)$	0.004	Ll. Amettler..., Phys. Lett. **B276**, 185 (1992)
χPT, $O(p^6) + ...$	0.42 ± 0.20	Ll. Amettler..., Phys. Lett. **B276**, 185 (1992)
χPT, $\mathcal{O}(p^6) + ...$	0.47	P. Ko, Phys. Lett. **B349**, 555 (1995)
χPT, ENJL	0.58 ± 0.03	S. Belluci..., Nucl. Phys. **B452**, 626 (1995)
χPT, ENJL	0.27 ± 0.18	J. Bijnens.., Phys. Lett. **B379**, 209 (1996)
VMD	0.30 ± 0.15	J. Ng .., Phys. Rev. D **46**, 5034 (1992)
Q box	0.70	J. Ng.., Phys. Rev. D **47**, 4937 (1993)
VMD	0.4	W. Allen.., Nuovo Cimento **A45**, 272 (1966)
χPT, $O(p^6)$	0.439 ± 0.09	M. Jetter, Nucl. Phys. **B459**, 283 (1996)
unitarized χPT	0.47 ± 0.10	E. Oset .., arXiv:hep-ph/0210283v1 21 Oct 2002

VMD: Vector Meson Dominance model

ENJL: Extended Nambu-Jona-Lasinio model

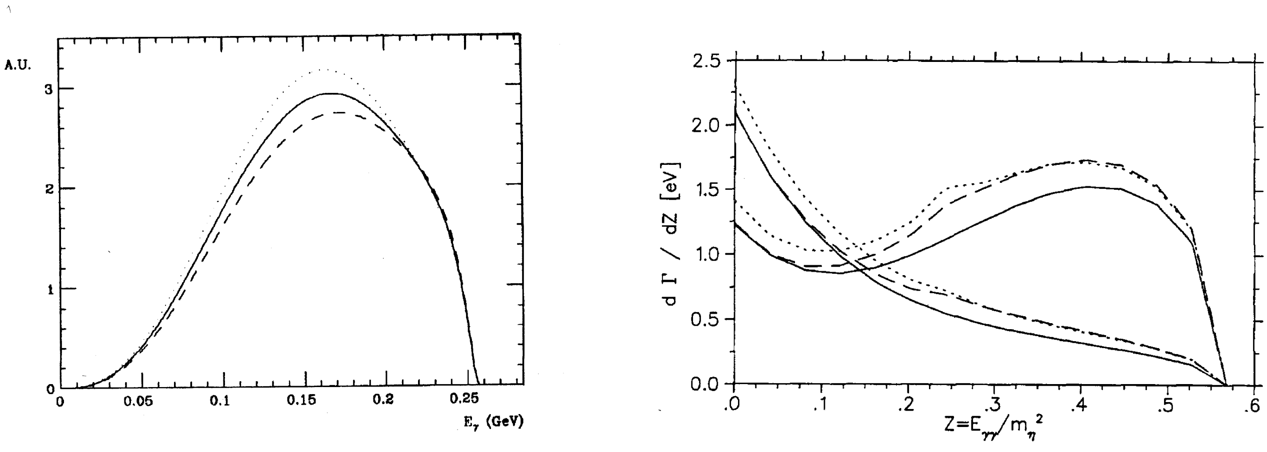

Figure 1: Examples of theoretical predictions for the $\eta \to \pi^0\gamma\gamma$ spectrum. Left figure from Ref. [3] and right figure from Ref. [15].

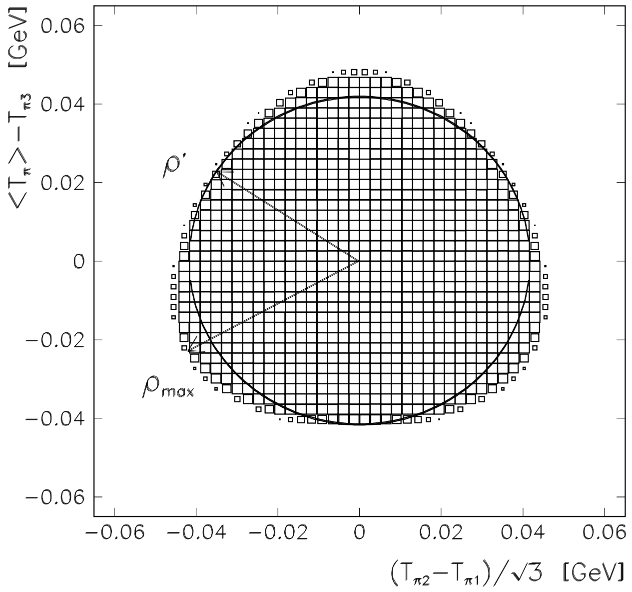

Figure 2: Symmetrized Dalitz plot for the decay $\eta \to 3\pi^0$. The plot is uniform along concentric circles. The deviation from a circular shape comes from using relativistic kinematics.

Table 3: Experimental measurements of the slope parameter α of the decay $\eta \to \pi^0\pi^0\pi^0$.

Experiment	Reference	α
Baglin *et al.*	[17]	-0.32 ± 0.37
GAMS-2000	[14]	-0.022 ± 0.023
Crystal Barrel	[18]	$-0.052 \pm 0.017 \pm 0.010$
SND	[19]	$-0.010 \pm 0.021 \pm 0.010$
Crystal Ball	[12]	-0.031 ± 0.004

the final state, we use a symmetrized Dalitz plot as illustrated in Fig. 2.
The deviation of the shape from a circle is the result of the use of relativistic kinematics. The experimental Dalitz plot density should be uniform in concentric circles around the center. A suitable variable to transform from the two-dimensional to a one-dimensional distribution is z:

$$z = 6\sum_{i=1}^{3}(E_i - m_\eta/3)^2/(m_\eta - m_{\pi^0})^2 = \rho^2/\rho^2_{max},$$

where E_i is the energy of the ith pion in the η rest frame and ρ is the distance from the center of the Dalitz plot. The variable z varies from $z = 0$, when all three π^0's have the same energy of $m_\eta/3$, to $z = 1$, when one π^0 is at rest.
The most accurate result comes from the very recent measurement by the Crystal Ball Collaboration at the AGS [12] yielding $\alpha = -0.031(4)$. The world data is summarized in Table 3.

Predictions based on chiral perturbation theory made by Kambor *et al.* [13], who used a dispersion calculation in which rescattering effects are treated to all orders, give values for α in the range $-(0.014 - 0.007)$. There is agreement with experiment in the sign but not in the magnitude. B. Holstein [16] has suggested the addition of a new dynamical input.
The proposed experiment has four times the event sample, plus double the acceptance. We expect the error to be reduced by half.

Table 4: The four classes of C, P, and T violations assuming CPT invariance.

Class	Violated	Valid
1	C, P, CT, PT	T, CP
2	C, P, T, CP, CT, PT	
3	P, T, CP, CT	C, PT
4	C, T, CP, PT	P, CT

Table 5: The seven classes of C, P, and T violations when CPT is not valid.

Class	Violated	Valid
1	C, CP, CT, CPT	P, T, PT
2	P, CP, PT, CPT	C, T, CT
3	T, CT, PT, CPT	C, P, CP
4	C, P, CP, CT, PT, CPT	T
5	C, T, CP, CT, PT, CPT	P
6	P, T, CP, CT, PT, CPT	C
7	C, P, T, CPT	

1.3 Search for New Physics by testing *CP* invariance

The Standard Model (SM) of the electroweak interactions has been phenomenally successful in giving a quantitative account of the various electroweak interactions. There is no evidence thus far for any failure. Yet the SM is not considered to be a theory. It needs 17 input parameters, not counting the neutrino sector. It does not explain such basic features as the existence of three families of fundamental fermions, the absence of the charge conjugates of the left-handed neutrinos and right-handed antineutrinos, the generation of widely different masses of the charged leptons, the origin of CP violation, etc. It is generally expected that the SM breaks down somewhere. A good place to look for New Physics appears to be the limit of validity of the basic symmetries of charge conjugation (C), parity (P), and time reversal (T), as well as CP and CPT in the different interactions.

If the CPT theorem holds, there are four distinct classes of the violations of C, P, and T (see Table 4). A particularly stringent limit on simultaneous P and T violation, class 3 of table 4, is obtained from the smallness of the limit on the electric dipole moment of the neutron. Yet, all classes, even #3, need better limits, in particular of electrostrong interactions of the quarks.

If CPT is not valid there are 7 main classes of C, P, T, and CPT violations (see Table 5).

CPT invariance is a fundamental theorem in quantum field theory, which results from locality and Lorentz invariance. Its validity in the context of quantum gravity is questionable [21]. String theory is intrinsically non-local, which could lead to a violation of CPT. Instantons give rise to charge-non-conserving transitions on the world sheet, and hence to CPT violations. Thus, the extent of the validity of CPT invariance must rest on experimental evidence which is either dynamic, such as in K^0 and η decay, or static, such as the equality of masses, half lives, magnetic moments, etc., of particles and their antiparticles. The most precise static test is $|m_{K^0} - m_{\overline{K^0}}|/m(\text{ave.}) < 10^{-18}$[7]. Testing CPT is extremely difficult in dynamic cases. The decay $\eta \to \pi^0\mu^+\mu^-$ provides a special opportunity. The decay spectrum must be even in $\cos\phi$, where ϕ is the angle between the π^0 and the μ^+ in the rest frame of the muon pair [22, 23].

CP symmetry means that the interaction of a set of left-handed particles is identical to the interaction of the complimentary set of right-handed antiparticles. The discovery of a 0.2% CP

violation in 1964 came as a great surprise. At the time of the discovery there were no theoretical models for CP violations, and the experimental upper limit for $K_L \to 2\pi$ was 0.3%! The origin of CP violation is still a mystery. There is widespread anticipation that detailed studies of CP violation may lead us to "New Physics" that goes beyond the Standard Model (SM). In the context of the SM, CP violation is described by the phase in the Cabibbo-Kobayashi-Maskawa quark-mixing matrix that is related to the existence of six quark flavors grouped into 3 families. CP violation shows up in family-changing interactions, while in family-conserving cases CP violation is unobservably small. The last prediction needs experimental verification which is lacking thus far. Of the 30 tests of CP listed in the Review of Particle Physics [7] only four are in this category: the 2π decays of the η and η' which at the present level are tests of P as well as CP.

Doable tests of CP invariance are hard to find for lack of mesons or di-fermion states which are eigenstates of the CP operator and have family-conserving interactions. There are several speculative ideas about unconventional CP violation such as spontaneous CP violation in the extended Higgs sector but they are hardly compelling.

The strong decay $\eta \to 2\pi$ is forbidden by CP and P invariance. An η can decay via the weak interaction; at the level of 10^{-7}, parity is not conserved any longer and for $BR \lesssim 10^{-7}$, $\eta \to 2\pi$ becomes a real test of CP invariance. There is considerable evidence that parity is conserved in purely strong and electromagnetic interactions. Strong-electroweak interference can produce a violation of P in carefully chosen processes, usually at a small level which is of order 10^{-6}.

The current experimental limits on the 2π decay of the η are:

$$BR(\eta \to \pi^0\pi^0) < 4.3 \times 10^{-4}[7]. \tag{1}$$

$$BR(\eta \to \pi^+\pi^-) < 3.3 \times 10^{-4}[7]. \tag{2}$$

Both limits have been obtained at a ϕ factory where η's are produced in the decay $\phi \to \eta\gamma$ ($BR = 1.2\%$).

There are no η beams, as the η lifetime is too short. The η's come from baryon decays, in particular from the $N(1535)\frac{1}{2}^-$ and $\Lambda(1670)\frac{1}{2}^-$ resonances, and from meson decays, observed e.g. in ϕ decay or in $\overline{p}p$ annihilation. In all cases there is plenty of 2π production; it is comparable to η production. Thus, it is a real experimental challenge to push the $\eta \to 2\pi$ limit down below the 10^{-7} level.

As indicated earlier, since $\eta \to 2\pi$ is a flavor-conserving interaction the expected BR in the SM is small. A recent calculation yielded $BR(\eta \to 2\pi) < 3 \times 10^{-17}$ [24]. The discovery of a much larger decay rate would be a sign for the existence of a nonconventional CP violating mechanism.

The upper limit for the CP test $\eta \to 2\pi$ is hard to improve appreciably with currently available setups because of the sizeable 2π background in every η production reaction. It is thus of interest to find another test. A novel possibility is $\eta \to 4\pi^0$ which is forbidden by CP and P. For η's produced in the reaction $\pi^- \, p \to \eta n$ near threshold there is no known background to $\eta \to 4\pi^0$. The chief drawback is the smallness of the final state phase space for $\eta \to 4\pi^0$ compared to $\eta \to 2\pi^0$.

The Crystal Ball Collaboration at the AGS has recently produced the first upper limit [25]:

$$BR(\eta \to 4\pi^0) < 6.9 \times 10^{-7}. \tag{3}$$

Combined with $\Gamma(\eta \to \text{all}) = 1.29 \pm 0.07\,\text{eV}$, this gives $\Gamma(\eta \to 4\pi^0) < 8.9 \times 10^{-4}$ eV. No events were found in a sample of 3×10^7 η decays produced near threshold in $\pi^- p \to \eta n$ close to threshold. To evaluate the sensitivity of this test, note that the η meson is an eigenstate of the CP operator. This allows for a comparison with a related but CP-allowed decay. The decay of a hypothetical η meson, the η_{hyp}, with $J^{PC} = 0^{++}$ into $4\pi^0$ is allowed. As η_{hyp} does not exist, we use instead $f_0(1500) \to 4\pi^0$. The f_0 has the same quantum numbers as the η except for its positive parity. The experimental value for the partial width is $\Gamma(f_0 \to 4\pi^0) = 33$ MeV. The ratio of the phase space compared to $\eta \to 4\pi^0$ is [26] $\Phi(\eta \to 4\pi^0)/\Phi(f_0 \to 4\pi^0) = 4.7 \times 10^{-8}$, so we might expect $\Gamma(\eta_{hyp} \to 4\pi^0) \simeq 1.6\,\mathrm{eV}$. Thus, the CP-violating amplitude for $\eta \to 4\pi^0$ compared to a comparable, allowed decay is

$$A_{\not{cp}}/A_{cp} < \left[\frac{8.9 \times 10^{-4}\,\mathrm{eV}}{1.6\,\mathrm{eV}}\right]^{\frac{1}{2}} = 2.3 \times 10^{-2} \tag{4}$$

at 90% CL.
A run of 1000 hours yielding over 2×10^8 etas would improve the current upper limit on the branching ratio by a factor of 20.

1.4 Three Tests of Charge Conjugation Invariance

C invariance, or charge conjugation symmetry, is the invariance of a system to the interchange of the colored quarks with their antiquarks of anticolor, the charged leptons with their antileptons, the left(right)-handed neutrinos with the left(right)-handed antineutrinos, and vice versa. According to QED and QCD, C invariance holds for all purely electromagnetic and all strong interactions, but the experimental limits are not impressive. The Review of Particle Physics [7] lists "all weak and electromagnetic decays whose observation would violate conservation laws." Seventeen tests of C invariance are listed: eight involve decays of the η, six of the η', two of the ω and one of the π^0. None has yielded a significant limit thus far [27]. Neither has the Pais test [28] which is the equality of any pair of C-symmetric reactions. Presented in ratio form, there is $R_1 = \sigma(\overline{p}p \to K^+X^-)/\sigma(\overline{p}p \to K^-X^+)$ and $R_2 = \sigma(\overline{p}p \to \pi^+Y^-)/\sigma(\overline{p}p \to \pi^-Y^+)$. The experimental data are $(R_1 - 1) < 2 \times 10^{-2}$ and $(R_2 - 1) < 1 \times 10^{-2}$ [29], which are not sufficiently precise for a useful analysis. The paucity of tests of C-invariance is related to the small number of eigenstates of the C-operator: only neutral, flavorless mesons such as η, η', and self-conjugate systems like $(p\overline{p})$, (e^+e^-), and $(K^0\overline{K^0})$ qualify.
According to Stuckelberg and Feynman, an antiparticle may be viewed as an ordinary particle that goes backward in time. This shows that the C, P, and T symmetries are interwoven. C reverses the sign of all additive quantum numbers of a particle but leaves its spin unaffected. Thus the C operator turns a left-handed neutrino into a left-handed antineutrino. The neutrinos studied in the lab all turn out to be left-handed and the antineutrinos right-handed, which means that there is full C violation of the weak interactions. The SM does not explain this; it merely is part of the input of the SM, namely it is assumed that all basic fermions come as left-handed doublets and right-handed singlets. This blatant asymmetry provides a strong impetus for making better experimental tests of the validity of C invariance.
Another argument which has kindled the interest in C is the experimental observations of the abundance of matter over antimatter in the universe, as well as photons over baryons: $n_B/n_\gamma < 10^{-10}$ [30]. In big-bang models of cosmology one naïvely expects the same abundance of matter and antimatter. The known CP violation is insufficient for explaining the experimental baryon/antibaryon asymmetry.
Finally, recent neutrino experiments have provided tantalizing hints of possible neutrino mixing and finite neutrino masses [31, 32].
The eta meson has the charge conjugation eigenvalue $C = +1$, and the $\pi^0\pi^0\gamma$ system with $J^P = 0^-$ has $C = -1$. Thus, the decay $\eta \to \pi^0\pi^0\gamma$ is strictly forbidden by C invariance. This decay would be an isoscalar electromagnetic interaction of hadrons. It has been suggested that there may exist an isotensor electromagnetic interaction with a C-violating component [33, 34]. The decay $\eta \to \pi^0\pi^0\gamma$ provides an opportunity to search for such an exotic interaction; it would be a clear signal for New Physics.

No searches for $\eta \to \pi^0\pi^0\gamma$ have been reported in the literature. A preliminary upper limit has been obtained using the Crystal Ball detector [35] from a sample of 1.9×10^7 η's. Candidate events in the signal region are predominantly ($\sim 85\%$) due to $\eta \to 3\pi^0$ decay with overlapping photon showers; the rest are due to $2\pi^0$ production with a split-off photon. The net yield is no events resulting in

$$BR(\eta \to \pi^0\pi^0\gamma) < 5 \times 10^{-4} \text{ at the 90\% C.L.} \tag{5}$$

This corresponds to $\Gamma(\eta \to \pi^0\pi^0\gamma) < 0.6\,\text{eV}$. To evaluate the sensitivity of this new result, we compare the upper limit with that of a C-allowed decay. In the absence of information on $f_0 \to \pi^0\pi^0\gamma$ — the f_0 is the preferred comparison state since it has $I^G(J^{PC}) = 0^+(0^{++})$ — we use $\rho \to \pi^+\pi^-\gamma$, with $BR = 1.0 \times 10^{-2}$, $\Gamma_\rho = 151$ MeV. In $\eta \to \pi^0\pi^0\gamma$ decay the $2\pi^0$ must be in a relative d-state. This implies that the decay goes by a magnetic quadrupole transition rather than a dipole as $\rho \to \pi^+\pi^-\gamma$ does, and we must include a reduction factor of order $(kR)^4$; we estimate this factor to be $(\frac{1}{2})^4 = 0.06$ at worst. After small adjustments for the difference in phase space and the Clebsch-Gordan factor, we find that a C-allowed decay has an expected decay width of 2×10^3 eV. This value is in reasonable agreement with two other estimates; the first is based on the allowed decay $\rho \to 2\pi$ and the other on the suppressed decay $\phi \to \pi^0\pi^0\gamma$. The sensitivity of the CB upper limit for $\eta \to \pi^0\pi^0\gamma$ is

$$A_{\not{C}}/A_C < \left[\frac{0.6\,\text{eV}}{2 \times 10^3\,\text{eV}}\right]^{\frac{1}{2}} = 1.7 \times 10^{-2}. \tag{6}$$

We are not aware of a more precise test of C invariance of an isoscalar interaction.

The radiative decay $\eta \to \pi^0\pi^0\pi^0\gamma$, is strictly forbidden by charge-conjugation invariance. No search for it has been published thus far. There are seven photons in the final state, which explains the need for a 4π acceptance detector. The background is mainly from $\eta \to 3\pi^0$ with either a split-off or an old photon shower from a previous interaction.

Recently a preliminary result, an upper limit, has been obtained using the Crystal Ball detector in the AGS experiment [20]:

$$BR(\eta \to \pi^0\pi^0\pi^0\gamma) < 7 \times 10^{-5}, \tag{7}$$

which corresponds to

$$\Gamma(\eta \to \pi^0\pi^0\pi^0\gamma) < 9.0 \times 10^{-2} \text{ eV, at the 90\% C.L.} \tag{8}$$

This is a test of an isovector electromagnetic interaction of hadrons. The sensitivity of this test has been evaluated using a similar approach as was used in the previous section. Starting from the strong decay of the ω-meson, $J^P = 1^-$, $\Gamma(\omega \to \pi^+\pi^-\pi^0) = 7.5$ MeV. We estimate the unknown radiative decay to be α times the strong decay width. Including an adjustment factor for the difference in phase space and the spin average weight factor [36] we obtain for an allowed $3\pi^0\gamma$ decay rate (if C invariance did not exist) 6.8×10^3 eV. The upper limit for a C-violating amplitude is thus

$$A_{\not{C}}/A_C \leq \left[\frac{9 \times 10^{-2}\,\text{eV}}{6.8 \times 10^3\,\text{eV}}\right]^{\frac{1}{2}} = 3.6 \times 10^{-3}. \tag{9}$$

This is the best upper limit for an isovector electromagnetic transition.

The decay of a neutral, flavorless, $C = +1$, pseudoscalar meson into three photons is forbidden rigorously by C-invariance. The 3γ decay should be small as it is a third order electromagnetic interaction and $\alpha^3 = 4 \times 10^{-7}$. The rate is further suppressed by substantial factors coming from phase space and angular momentum barrier considerations [37]. The decay $\eta \to 3\gamma$ can be isoscalar or isovector and even the hypothetical isotensor interaction. The Particle Data Group [7] lists the upper limit for the $\eta \to 3\gamma$ branching ratio as 5×10^{-4}.

The Crystal Ball experiment at the AGS has produced a new, still preliminary result which is [20, 38]

$$BR(\eta \to 3\gamma) < 1.8 \times 10^{-5} \tag{10}$$

at the 90% C.L. Using Eq. 4, this corresponds to

$$\Gamma(\eta \to 3\gamma) < 2.3 \times 10^{-2}\,\mathrm{eV}. \tag{11}$$

The largest background in this experiment is from $\eta \to 3\pi^0 \to 6\gamma$ decay, $BR(\eta \to 3\pi^0) = 0.32$, when photon showers overlap in the detector. The background from $\eta \to \pi^0\gamma\gamma$ decay when two photons overlap is insignificant becauser of the smallness of the branching ratio, $BR(\eta \to \pi^0\gamma\gamma) = 3 \times 10^{-4}$. The background from $\eta \to 2\gamma$ with two split-offs is greatly suppressed in our analysis.

There is no straightforward way to assess the sensitivity of the $\eta \to 3\gamma$ process analogous to the one used in the preceeding two sections. The triplet positronium state decays into 3γ but it has $J^{PC} = 1^{--}$, which is not suitable for the task at hand.

The decay $\eta \to 3\gamma$ can take place by the allowed, C-violating, CP conserving weak interaction of the SM denoted by $BR(\eta \to 3\gamma)_w$. Using a quark-loop model Dicus [39] has obtained

$$BR(\pi^0 \to 3\gamma)_w = (1.2 \times 10^{-5})\frac{\alpha}{(2\pi)^5}G^2 m_\pi^4 \left(\frac{m_\pi}{m}\right)^8, \tag{12}$$

where G is the Fermi constant and m is an effective quark mass. Choosing $m > (1/7)m_N$ this yields

$$BR(\pi^0 \to 3\gamma) < 6 \times 10^{-19}. \tag{13}$$

A similar expression for the allowed weak decay of the η yields for $m > (1/5)m_N$ the limit [40]

$$BR(\eta \to 3\gamma)_\omega \quad < 3 \times 10^{-12}. \tag{14}$$

$\eta \to 3\gamma$ decay due to a CP-violating new interaction is not likely. P. Herczeg [41] has shown that in renormalizable gauge models with elementary quarks the flavor conserving nonleptonic interactions of the quarks do not contain in first order a P-conserving CP-violating component. The CP-violating contributions to $BR(\eta \to 3\gamma)$ in such models are therefore negligible relative to $BR(\eta \to 3\gamma)_\omega$. P. Herczeg [40] has considered the existence of a flavor conserving C- and CP-violating interaction ($\bar{H}$). Using the stringent limits imposed by the upper limit of an electric dipole moment of the neutron, he obtains

$$B(\eta \to 3\gamma)_{\bar{H}} < 10^{-19}. \tag{15}$$

It is of interest to compare the relative sensitivity of the decay rates of the three lightest pseudoscalar mesons into 3γ. The simplest effective Hamiltonian for a 0^- meson decaying into 3γ contains seven derivatives [40], consequently,

$$BR \sim m_{0^-}^{12} \cdot \Gamma(0^- \to all). \tag{16}$$

This results in the following sensitivity comparison

$$BR(\eta \to 3\gamma) \ : \ BR(\pi^0 \to 3\gamma) \ : \ BR(\eta' \to 3\gamma) = 1 : 10^{-5} \ : \ 5. \tag{17}$$

Thus, even though the experimental limit for $BR(\pi^0 \to 3\gamma) < 3 \times 10^{-8}$ is small, it is 100 times less sensitive than the experimental upper limit for $\eta \to 3\gamma$ given in Eq. 10. On the other hand, $BR(\eta' \to 3\gamma) < 1 \times 10^{-4}$ quoted in Ref. [7] is a comparable test of C.

In recent years there has been a growing interest in quantum field theory over noncommutative spaces in part because of the connection to string theories. Noncommutative CP-violating effects are now being estimated. They may actually dominate over the SM contribution [42].

Grosse and Liao [43] have shown that a generalization of the anomalous $\pi^0 \to 2\gamma$ interaction can induce a C-violating $\pi^0 \to 3\gamma$ amplitude in noncommutative quantum electrodynamics. The prediction for $BR(\pi^0 \to 3\gamma) = 6 \times 10^{-21}$ is still far from the experimentally reachable level but it shows that there are new options. It will be interesting to see what noncommutative field theory will predict for $\eta \to 3\gamma$.

2 Experimental details

2.1 Experimental apparatus

The proposed detector for a high quality measurement of the decay ratio and spectral shape of $\eta \to \pi^0\gamma\gamma$ is the Crystal Ball multiphoton spectrometer at MAMI-B or MAMI-C in Hall A2 of MAMI using the Glasgow photon tagger. The CB is augmented with TAPS as a forward detector, with a charged particle tracker which consists of two DAPHNE coaxial cylindrical multiwire proportional chambers (MWPC) and with a particle identification detector (PID) which is a cylinder made of 24 scintillator strips 2 mm thick located around the liquid H_2 target, see Ref. [44] for details on the experimental setup. The experimental apparatus provides close to 4π sr coverage for outgoing photons. Protons will be detected by the TAPS forward wall for $\Theta_{lab} < 21°$, and by the MWPC plus PID for other angles. The acceptance calculated for example for the $\eta \to \pi^0\gamma\gamma$ is about 30% when all five particles are detected.
It is of some interest to compare the expected performance of CB@MAMI with the one of CB@AGS. There are some advantages of CB@MAMI over CB@AGS:

1. CB@MAMI has new electronics with a TDC and a flash ADC for every crystal. This will minimize the effects of old tracks on the event efficiency and improve the photon energy resolution because of the "afterglow" in the NaI can be corrected for.

2. The forward tunnel in the CB is covered by TAPS which increases the acceptance for photons by about $(0.94)^4 = 28\%$.

3. The PID is only 2 mm thick and 32 cm long. The veto barrel from the CB@AGS was 5 mm thick and 120 cm long. We expect that photon conversion in the vacuum pipe and PID will be typically 2% resulting in a gain in acceptance of 15%.

4. The new liquid H_2 target is only 5 cm long instead of 10 cm, which improves the π^0 identification and decreases the background.

5. At MAMI we will have tagged etas; the production process is $\gamma p \to \eta p$. At the AGS we had "selftagging" using $\pi^- p \to \eta n$.

6. The pion beam at the AGS, like all secondary beams, has a halo of scattered and decayed particles as well as a useless electron component. The MAMI beam is well collimated and clean. On the other hand, the AGS experiment used the incident pion in the trigger which is not easily done at MAMI with the incident photon. The worst problem is in the presence of many low energy photons in the beam because the photon beam has a bremsstrahlung spectrum.

7. The duty factor at the AGS was 50%; it is about 100% at MAMI.

2.2 Event rate

For the proposed experiment the parameters entering the count rate estimate and the resulting beam time request are:

- Incoming electron beam energy: $E_0 = 882$ MeV.
- Tagged photon energy range: $E_\gamma^t = 720 - 820$ MeV, thus $\Delta E_\gamma = 100$ MeV.
- Electron count rate in the tagger: $N_e = 5 \times 10^5 \frac{1}{s} \frac{1}{MeV}$.
- Tagging efficiency: $\varepsilon_t \approx 50\%$.
- Tagged photon flux: $N_\gamma = 2.5 \times 10^5 \frac{1}{s} \frac{1}{MeV}$.
- Number of protons in a 5 cm long LH_2 target (modified DAPHNE cryo target): $N_t = 2.1 \times 10^{23} \frac{1}{cm^2}$.
- Eta photoproduction cross section: $\sigma_t(\gamma p \to \eta p) = 14 \mu b$

The number of etas is

$$N_\gamma \Delta E_\gamma \Delta t N_t \sigma_t = 2.6 \times 10^5 \, \eta/h.$$

With a detection efficiency for the $\pi^0\gamma\gamma$ channel conservatively taken to be 20%, a data acquisition system livetime of 70%, and $BR(\eta \to \pi^0\gamma\gamma) = 3 \times 10^{-4}$, we expect 11 good $\eta \to \pi^0\gamma\gamma$ events each hour. In 200 hours, we will get 2200 events. We estimate needing 40 hours of empty target data for background measurements, and 30 hours for trigger studies. At the AGS, the CB recorded 500 events. The new run at MAMI is planning a four times larger sample with a commensurate improvement in the results. It will show definitively who is right, GAMS 2000 or CB@AGS.

3 Summary

In the preceeding section we have presented the status of six especially interesting eta meson decays and we have indicated the significance of an eta program with a tenfold increase in sensitivity over CB@AGS. The feasibility of the experiment with the CB as the central detector has been amply demonstrated by the success of the CB@AGS, see for example Refs. [11, 12, 36]. The added features, new electronics, TAPS, tracker, and PID, help in further suppressing background and increasing efficiency.

The decay rate and spectrum for $\eta \to \pi^0\gamma\gamma$ are the most important and should be measured in 2004. It is labeled Phase I.

We like to have a 1000 hours run for Phase II to set record limits on C and CP invariance in various forbidden η decays.

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