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Mainz Microtron MAMI

Collaboration A2: "Tagged Photons"
Spokesperson: A. Thomas

Update of Proposal for an Experiment

"Test of Chiral Perturbation Theory and C and CP Invariance in η Decay"

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Abstract of Physics :

We propose to investigate six neutral decay modes of the eta meson; all are measured simultaneously. This is the second phase of the experiment with the MAMI-C beam. The first stage was successfully completed in 2007. Our priority is the measurement of the Dalitz plot and decay spectrum of $\eta \to \pi^0\gamma\gamma$. The decay amplitude is determined by the third-order term in the momentum expansion; the first term is zero and the second is small. Thus, $\eta \to \pi^0\gamma\gamma$ provides a unique, sensitive test of Chiral Perturbation Theory (χPT). Furthermore we will measure Dalitz plot of $\eta \to 3\pi^0$ to investigate the speculated quadratic parameter for the slope in $\eta \to 3\pi^0$ and the cusp at the opening of $\pi^0\pi^0 \to \pi^+\pi^-$. We will also improve by a factor of $10 - 20$ three tests of charge conjugation invariance, namely $\eta \not\to 2\pi^0\gamma$, $\eta \not\to 3\pi^0\gamma$, and $\eta \not\to 3\gamma$, and improve on a unique test of CP invariance, namely $\eta \not\to 4\pi^0$. Etas are photoproduced in the reaction $\gamma p \to \eta p$ with tagged photons of 707 to 1450 MeV.

Abstract of Equipment :

We require a beam of tagged photons incident on a liquid-hydrogen target. The detector is the 4π Crystal Ball photon spectrometer in combination with TAPS as forward wall, and a scintillator particle identification detector (PID). The Glasgow-Mainz tagging system will provide the intense photon beam.

MAMI Specifications :

beam energy	1558 MeV
beam current	< 20 nA
beam polarisation	unpolarized (circular polarization is acceptable)

Photon Beam Specifications :

tagged energy range	600 - 1480 MeV
photon beam polarisation	unpolarized (circular polarization is acceptable)

Equipment Specifications :

detectors	Crystal Ball/TAPS/PID
target	10 cm liquid hydrogen

Beam Time Request :

set–up/tests with beam	50 hours
data taking	900 hours (parallel with other hydrogen-target experiments)

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1 Introduction

The η is a unique meson because it provides a very sensitive test of Chiral Perturbation Theory, χPT. The decay rate and the Dalitz plot of $\eta \to \pi^0\gamma\gamma$ are determined by the third order term of χPT. Another test of χPT is provided by the slope parameter of $\eta \to 3\pi^0$ and the cusp in the $\pi^0\pi^0$ invariant mass at the opening of $\pi^0\pi^0 \to \pi^+\pi^-$. Selected eta decays such as $\eta \to 3\gamma$, $\eta \to 2\pi^0\gamma$, and $\eta \to 3\pi^0\gamma$ are forbidden to occur by charge conjugation invariance of the flavor conserving electro-strong interaction. A sensitive search for those forbidden η decays gives important new limits on C-invariance. Finally, the decay mode $\eta \to 4\pi^0$ is forbidden by CP-invariance.

The importance of the six η decays was discussed in details in our 2003 MAMI-B and 2005 MAMI-C proposals [1, 2]. Here we give an update of the experimental situation in η decay physics.

In the CB experiment at the AGS the above six eta decays together with several others were successfully studied [3, 4, 5, 6, 7]. They were investigated simultaneously which resulted in a substantial savings in running and analyzing time. In a 300 hour eta run in 2004 at MAMI-B about 3×10^7 η's were produced on target. The results on the slope parameter of the $\eta \to 3\pi^0$ Daltz plot obtained in 2004 with MAMI-B were recently published [8]. The first A2 collaboration MAMI-C publication was also dedicated to the physics of the $\eta \to 3\pi^0$ decay [9]. The high statistics, high accuracy data obtained with the Crystal Ball at MAMI-C allow us to investigate such fine effects as the cusp in the $\pi^0\pi^0$ invariant mass at the opening of $\pi^0\pi^0 \to \pi^+\pi^-$. Some results on the cusp are shown in Ref. [9]. More data are needed to investigate this effect in detail.

71.6% of all η decays result in neutral particles — photons and π^0's. The neutral decay modes are listed in Table 1, which also shows the physics theories and symmetries which can be investigated with each mode.

Table 1: The Neutral η Decays.

Decay Mode	Branching Ratio	Physics highlight
All Neutrals	$(71.91 \pm 0.34)\%$	
2γ	$(39.31 \pm 0.20)\%$	$SU(3)$ octet-singlet mixing
$3\pi^0$	$(32.56 \pm 0.23)\%$	$\chi PTh;\ m_u - m_d$
$\pi^0\gamma\gamma$	$(4.4 \pm 1.5) \times 10^{-4}$	$\chi PTh,\ O(p^6)$
$2\pi^0$	$< 3.5 \times 10^{-4}$	P and CP invariance
$4\pi^0$	$< 6.9 \times 10^{-7}$	P and CP invariance
$\pi^0\pi^0\gamma$	$< 5 \times 10^{-4}$	C (isoscalar) invariance
$\pi^0\pi^0\pi^0\gamma$	$< 6 \times 10^{-5}$	C (isovector) invariance
3γ	$< 1.6 \times 10^{-5}$	C (isovector, isoscalar)
4γ	$< 2.8 \times 10^{-4}$	
$\pi^0\pi^0\gamma\gamma$	$< 1. \times 10^{-3}$	χPTh, New Physics
$\nu_e\bar{\nu}_e$	$< 2.8\%$	New Physics (leptoquarks)
$\nu_e\bar{\nu}_\mu$	$< 2.8\%$	New Physics (leptoquarks)
$\nu_e\nu_e$	$< 2.8\%$	New Physics (leptoquarks)
$\gamma\nu\nu$	$< 2.8\%$	New Physics (leptoquarks)
$\pi^0\nu\bar{\nu}$	$< 2.8\%$	New Physics (leptoquarks)

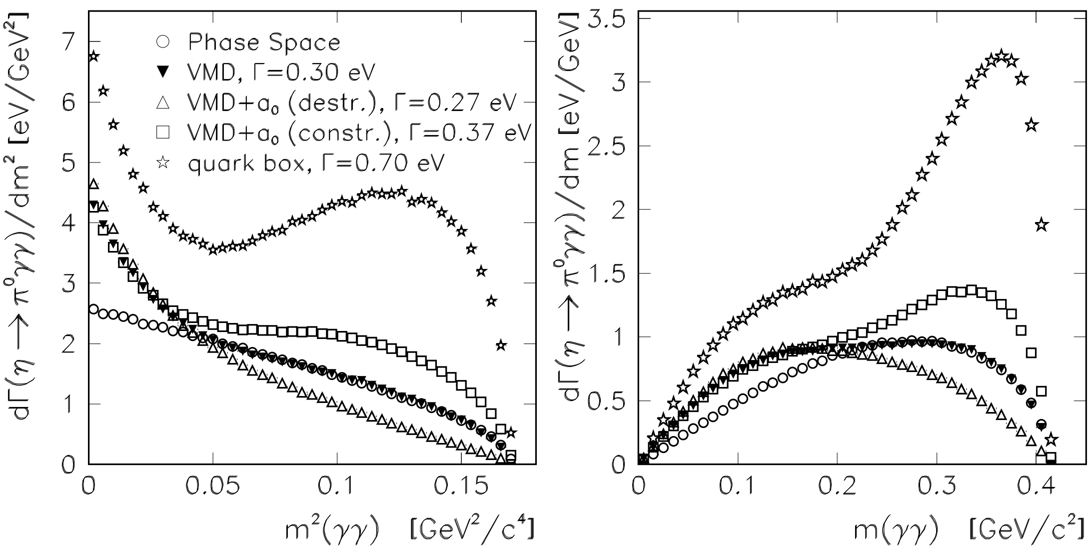

Figure 1: Comparison of the χPT calculations of Refs. [14, 15] showing the dependence of the $\eta \to \pi^0\gamma\gamma$ decay width on $m^2(\gamma\gamma)$ (left) and on $m(\gamma\gamma)$ (right).

2 Neutral η decays

2.1 The $\eta \to \pi^0\gamma\gamma$ decay

The rare, doubly-radiative decay

$$\eta \to \pi^0\gamma\gamma \tag{1}$$

has attracted much attention as there are large uncertainties in the experimental determination of its decay width and in the calculations of it using χPT. The uncertainties in χPT calculations of the amplitude for the $\eta \to \pi^0\gamma\gamma$ transition are related to the fact that the leading term $\mathcal{O}(\mathsf{p}^2)$ and the tree contributions at $\mathcal{O}(\mathsf{p}^4)$ are zero as neither π^0 nor η can emit a photon. The pion and kaon loops at $\mathcal{O}(\mathsf{p}^4)$ are greatly suppressed due to G-parity invariance and the large mass of the kaons, respectively. The main contribution to the $\eta \to \pi^0\gamma\gamma$ decay amplitude comes from the $\mathcal{O}(\mathsf{p}^6)$ counterterms that are needed in χPT to cancel various divergences. The coefficients of these counterterms are not determined by χPT itself; they depend on the model used for the calculation. As the $\eta \to \pi^0\gamma\gamma$ decay has a three-body final state, its Dalitz plot reflects the decay amplitude. Thus, a complete test of χPT with its $\mathcal{O}(\mathsf{p}^6)$ chiral coefficients requires an experimental measurement of both the $\eta \to \pi^0\gamma\gamma$ decay rate and the Dalitz plot. So far this has not been done.
The experimental challenges of measuring $\eta \to \pi^0\gamma\gamma \to 4\gamma$ are formidable because of the smallness of doubly-radiative processes, which is typically of order $\alpha^2 = 1/137^2$. In practice, it requires the suppression of large backgrounds and the subtraction of the remaining background contributions. Major backgrounds, which can mimic $\eta \to \pi^0\gamma\gamma$ events, come from $\eta \to 3\pi^0 \to 6\gamma$ decays with electromagnetic showers that overlap in the photon detector and also from $\eta \to \gamma\gamma$ decays with split-off showers. As $BR(\eta \to 3\pi^0) = 0.325$ and $BR(\eta \to \gamma\gamma) = 0.394$, the background from these η decay modes is usually significant for all types of photon detectors. In the experiments where η mesons are produced from decays of baryon states, the largest contribution to the four-photon final state comes from $\pi^0\pi^0$ production. Therefore, this process must be suppressed substantially during the analysis in order to see a tiny signal from $\eta \to \pi^0\gamma\gamma$. This is

even more important for a measurement of the $\eta \to \pi^0\gamma\gamma$ Dalitz plot needed for understanding the decay amplitude. Since the density of the event distribution across the $\eta \to \pi^0\gamma\gamma$ Dalitz plot varies depending on the model used in the χPT calculations, a good experimental acceptance for the full Dalitz plot is also essential.

The history of early attempts to measure and calculate the $\eta \to \pi^0\gamma\gamma$ decay have been reviewed in detail in Ref. [10]. A major advance was made in 1981 with the GAMS experiment [11, 12], which used a forward wall of 1400 Cerenkov counters that provided good energy and spatial resolution for high-energy photons. 6×10^5 η mesons were produced in the $\pi^- p \to \eta n$ reaction, improving the statistics compared to previous experiments by two orders of magnitude. A narrow peak of 40 events in the $\pi^0\gamma\gamma$ invariant-mass spectrum at the mass of the η meson was interpreted as the $\eta \to \pi^0\gamma\gamma$ signal. Much attention was paid to suppressing the $\eta \to 3\pi^0$ background. In 1982, the GAMS collaboration reported that $BR(\eta \to \pi^0\gamma\gamma) = (9.5 \pm 2.3) \times 10^{-4}$ [11]. A better understanding of the $\eta \to 3\pi^0$ background resulted in a smaller value, $BR(\eta \to \pi^0\gamma\gamma) = (7.1 \pm 1.4) \times 10^{-4}$, published in 1984 [12]. No estimate of the remaining $\eta \to 3\pi^0$ background among the $\eta \to \pi^0\gamma\gamma$ candidates was presented.

For two decades, the revised GAMS result, $\Gamma(\eta \to \pi^0\gamma\gamma) = 0.84 \pm 0.17$ eV [12], was the favored experimental value for this decay width. It brought much interest to theoretical calculations that tried to reproduce the surprisingly large $\eta \to \pi^0\gamma\gamma$ decay width. According to Ametller *et al.* [13], the decay amplitude based only on vector-meson dominance (VMD) yields $\Gamma(\eta \to \pi^0\gamma\gamma) = 0.31$ eV. Including the pion and kaon loops at $\mathcal{O}(\mathsf{p}^4)$ and $\mathcal{O}(\mathsf{p}^8)$ increases the width to 0.42 eV. Finally, adding a_0- and a_2-meson exchange, with the assumption of constructive interference with vector mesons, results in a width of 0.50 eV. This is only about half the experimental value for $\Gamma(\eta \to \pi^0\gamma\gamma)$. Similar VMD results have been obtained by Ng and Peters [14]: $\Gamma_{\text{VMD}}(\eta \to \pi^0\gamma\gamma) = 0.30^{+0.16}_{-0.13}$ eV and $\Gamma_{\text{VMD}+a_0}(\eta \to \pi^0\gamma\gamma) = 0.37^{+0.23}_{-0.17}$ eV. The same authors increase the predicted $\eta \to \pi^0\gamma\gamma$ decay width to 0.70 eV in a study based on the quark-box diagram [15]. Ko in Ref. [16] revised the calculation of Ref. [13] by including contributions of C-odd axial-vector resonances; his result is $\Gamma(\eta \to \pi^0\gamma\gamma) = 0.47 \pm 0.20$ eV. Jetter in Ref. [17], using two different models, obtained $\Gamma_{L^6+\mathcal{O}(\mathsf{p}^4)+L^4+\text{fact.}}(\eta \to \pi^0\gamma\gamma) = 0.77 \pm 0.16$ eV and $\Gamma_{\text{VMD+loops}}(\eta \to \pi^0\gamma\gamma) = 0.44 \pm 0.09$ eV. The study of the $\eta \to \pi^0\gamma\gamma$ decay via the quark-box diagram in the three-flavor Nambu-Jona-Lasinio model by Nemoto *et al.* [18] resulted in 0.92 eV for the decay width. From the overview of the existing calculations, one can see that only the calculations based on the quark-box diagram get close to the GAMS result for $\Gamma(\eta \to \pi^0\gamma\gamma)$.

After 2001, the experimental situation on measuring the $\eta \to \pi^0\gamma\gamma$ decay changed greatly. New experiments reported decay-width values which were two to three times smaller than the GAMS result and were in better agreement with χPT calculations. The Crystal Ball (CB) collaboration at the AGS conducted an experiment devoted to investigations of rare η-meson decays with a total of 2.8×10^7 η mesons produced in the $\pi^- p \to \eta n$ reaction near threshold. The analysis of the data was performed in a few stages. The latest value is $BR(\eta \to \pi^0\gamma\gamma) = (2.21 \pm 0.24_{\text{stat}} \pm 0.47_{\text{syst}}) \times 10^{-4}$ [6]. An independent analysis [19] of the same CB data yielded the relative branching ratio $B_1 = (8.3 \pm 2.8_{\text{stat}} \pm 1.4_{\text{syst}}) \times 10^{-4}$ with respect to $BR(\eta \to 3\pi^0)$; this implies $BR(\eta \to \pi^0\gamma\gamma) = (2.7 \pm 0.9_{\text{stat}} \pm 0.5_{\text{syst}}) \times 10^{-4}$. Meanwhile, the SND collaboration at VEPP-2M reported $BR(\eta \to \pi^0\gamma\gamma) = (2.1^{+3.8}_{-1.9}) \times 10^{-4}$ [10]. However, the signal was just $7.0^{+12.9}_{-6.5}$ events.

The most recent calculations of $\Gamma(\eta \to \pi^0\gamma\gamma)$ resulted in 0.47 ± 0.10 eV [21] and 0.45—0.53 eV [22], showing good agreement with the latest experimental values, like $\Gamma(\eta \to \pi^0\gamma\gamma) = 0.45 \pm 0.12$ eV from Ref. [20]. Surprisingly low, in comparison with all earlier measurements and the latest calculations, is the recent preliminary result of the KLOE collaboration [23], $BR(\eta \to \pi^0\gamma\gamma) = (0.84 \pm 0.27_{\text{stat}} \pm 0.14_{\text{syst}}) \times 10^{-4}$ which is based on a signal of 68 ± 23 events. From the theoretical point of view, a small decay width like this could be the result of destructive interference between the vector-meson and other meson contributions. To check this hypothesis experimentally, one must investigate the density of the $\eta \to \pi^0\gamma\gamma$ Dalitz plot, which reflects the

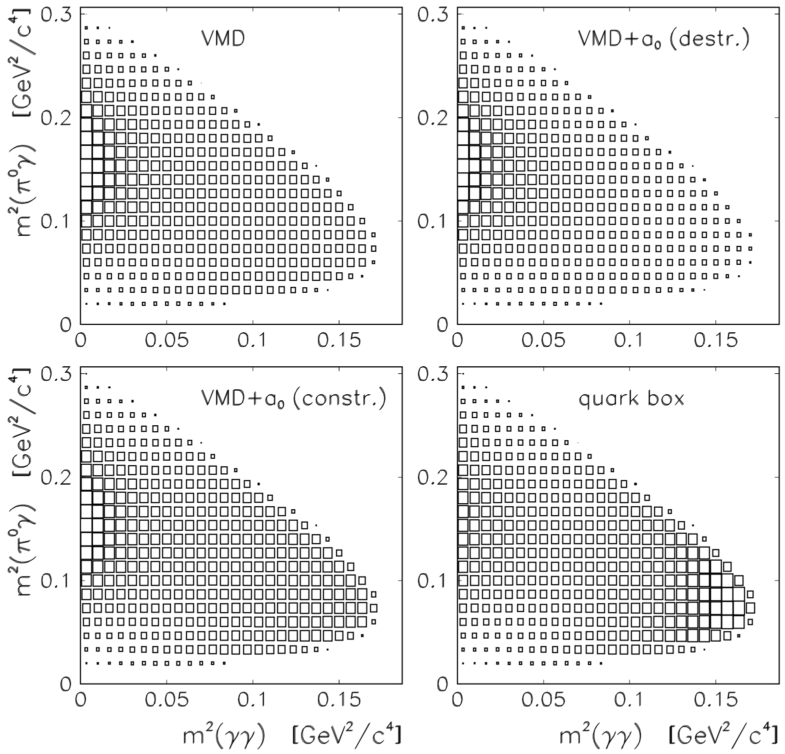

Figure 2: The $\eta \to \pi^0\gamma\gamma$ Dalitz plot for the χPT calculations of Refs. [14, 15].

decay amplitude.
One can see that the existing experimental results and theoretical calculations for $\Gamma(\eta \to \pi^0\gamma\gamma)$ vary a lot. Also, strictly speaking, the agreement between the measured and calculated decay width is not sufficient to prove χPT calculations. Since every calculation of $\Gamma(\eta \to \pi^0\gamma\gamma)$ makes a specific prediction for the decay Dalitz plot, the experimental measurement of this plot must confirm the theoretical prediction, too. χPT calculations also depict the $d\Gamma(\eta \to \pi^0\gamma\gamma)$ dependence on the two-photon invariant mass, $m(\gamma\gamma)$, (or the invariant mass squared, $m^2(\gamma\gamma)$) in the $\eta \to \pi^0\gamma\gamma$ decay. In Fig. 1, we illustrate the predictions for both the $m(\gamma\gamma)$ and $m^2(\gamma\gamma)$ spectra, which are obtained from the decay amplitudes described in detail in Refs. [14, 15]. The prediction based on the vector-meson contribution alone gives the basic decay width and two-photon invariant-mass spectrum that is close to phase space in the region where $m^2(\gamma\gamma) > 0.05$ GeV$^2/c^4$. Note that the "pure" VMD prediction for $d\Gamma(\eta \to \pi^0\gamma\gamma)/dm(\gamma\gamma)$ is similar for most of the existing calculations [13, 14, 16, 17, 21]. Adding other contributions to the vector-meson part, for example a_0-meson exchange, changes the decay width and the invariant-mass spectrum, which depend on the sign of the interference term. As shown in Fig. 1, there is a typical correlation between the change of the decay width and the change in the two-photon invariant-mass spectrum. Evidently, increasing the total decay width occurs mostly due to the rise in the $d\Gamma(\eta \to \pi^0\gamma\gamma)$ spectrum at high $m(\gamma\gamma)$ values. The corresponding $\eta \to \pi^0\gamma\gamma$ Dalitz plots are illustrated in Fig. 2. Thus far, none of the experiments has presented a reliable measurement of the $\gamma\gamma$ invariant-mass spectrum, which is needed to provide a unique test of χPT calculations and to obtain the information necessary for determining the coefficients of the $\mathcal{O}(\mathsf{p}^6)$ counterterms.
The first results for the $d\Gamma(\eta \to \pi^0\gamma\gamma)/dm^2(\gamma\gamma)$ distribution and a new value for the $\eta \to \pi^0\gamma\gamma$ branching ratio were presented recently by the CB@AGS collaboration [6]. The statistics of

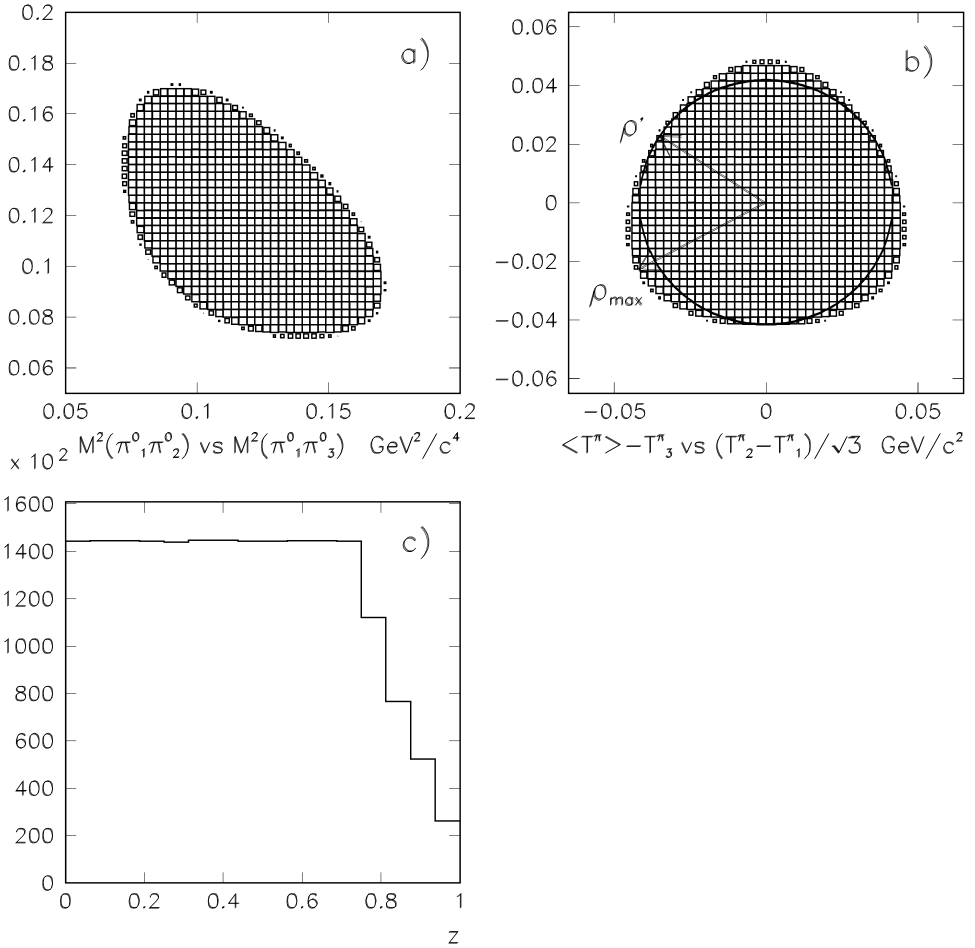

Figure 3: Distributions for the phase-space decay of $\eta \to 3\pi^0$ (i.e., when $\alpha = 0$) obtained from a Monte Carlo simulation: (a) Dalitz plot showing $M^2(\pi^0_1\pi^0_2)$ versus $M^2(\pi^0_1\pi^0_3)$, (b) Dalitz plot where T^π_i is the kinetic energy of each of the three pions, and $\langle T^\pi \rangle$ is the mean kinetic energy of the three pions (all energies are calculated in the η rest frame); (c) variable $z = \rho^2/\rho^2_{\rm max}$, reflecting the density of the Dalitz plot.

the AGS experiment is not sufficient to produce an informative Dalitz plot. The proposed high statistic experiment will measure the decay spectrum and the Dalitz plot as well as the branching ratio of the decay.

2.2 π^0 slope parameter in $\eta \to 3\pi^0$ decay

The experimental study of the simple and pure strong-interaction reaction

$$\pi^0\pi^0 \to \pi^0\pi^0 \tag{2}$$

is a real challenge as neither a π^0 target nor a π^0 beam is available. The properties of reaction (2) can be extracted indirectly from complicated processes, for example, from $K^+ \to \pi^0\pi^0 e^+\nu_e$ (K^+_{e4}) that is the weak decay of the K^+ followed by strong final-state interactions between the two π^0s. Major disadvantages of studying reaction (2) in K^+_{e4} are its small branching ratio (2.2×10^{-5}) and the complications from the four complex form factors for the K^+_{e4} decay amplitude needed to describe the four-particle final state. Another process that can be used for the indirect study of reaction (2) is the decay

$$\eta \to 3\pi^0 \; , \tag{3}$$

where the $\pi^0\pi^0$ final-state interaction can be seen in a difference of the $\eta \to 3\pi^0$ decay amplitude from phase space. The experimental study of this decay has several major advantages: the relatively large branching ratio for $\eta \to 3\pi^0$ (32.5%), a high yield of η mesons in many production reactions, very small background from other $3\pi^0$ contributions, especially in η production close to the threshold. Due to the low energies of the decay π^0s, $\pi^0\pi^0$ rescattering in $\eta \to 3\pi^0$ is expected to be dominated by S-wave. This leads to the parametrization of the $\eta \to 3\pi^0$ decay amplitude as $A(\eta \to 3\pi^0) \sim 1 + \alpha z$, where α is the quadratic slope parameter that describes the difference from phase space. A convenient definition of the kinematic variable z is

$$z = 6\sum_{i=1}^{3}(E_i - m_\eta/3)^2/(m_\eta - 3m_{\pi^0})^2 = \rho^2/\rho^2_{\max} \; , \tag{4}$$

where E_i is the energy of the i^{th} pion in the η rest frame, and ρ is the distance from the center of the $\eta \to 3\pi^0$ Dalitz plot. The variable z varies from 0, when all three π^0s have the same energy of $m_\eta/3$, to 1, when one π^0 is at rest. A geometrical interpretation of equation (4) gives $z = 0$ when $\rho = 0$ and $z = 1$ when $\rho = \rho_{\max}$. The density of the $\eta \to 3\pi^0$ Dalitz plot is described by $|A(\eta \to 3\pi^0)|^2 \sim 1 + 2\alpha z$. The phase-space decay of $\eta \to 3\pi^0$ (i.e., when $\alpha = 0$) gives a uniform density of the Dalitz plot, which is shown in Fig. 3(b). The corresponding distribution of the variable z is shown in Fig. 3(c); it is uniform for z from 0 to $\approx$0.75 . Experimentally, the slope parameter α is usually determined from the deviation of the measured z distribution from the corresponding distribution obtained by a Monte Carlo simulation in which the $\eta \to 3\pi^0$ decay amplitude is independent of z.

The $\eta \to 3\pi^0$ decay, which violates G-parity, occurs mostly because of the u-d quark mass difference. The precision measurement of the $\eta \to 3\pi^0$ decay width, $\Gamma(\eta \to 3\pi^0) \sim (m_d - m_u)^2(1 + 2\alpha z)$, and the parameter α are important tests of χPT. In the χPT momentum expansion in orders of p, the leading $\mathcal{O}(\mathsf{p}^2)$ term of the decay amplitude explicitly depends on $m_d - m_u$. However, including this term and the second-order counter terms, $\mathcal{O}(\mathsf{p}^4)$, is not sufficient [25] to yield a decay width that is close to the measured value of 423 eV [26]. The use of dispersion relations [27, 28], which include pion rescattering to all orders, partially improves the agreement with the experimental value. For the parameter α, the dispersion-relation calculations of Ref. [27] give a negative value in the range -0.007 to -0.0014, depending on the assumptions made. The results of these calculations are outside the value, $\alpha = -0.031 \pm 0.004$, adopted by the Particle Data Group (PDG) [26]. This value for α is based on the analysis of 0.9×10^6 $\eta \to 3\pi^0$ decays measured by the Crystal Ball at the AGS [29]. The most recent calculation with a chiral effective Lagrangian within the $U(3)$ framework [30] yields $\alpha = -0.031 \pm 0.003$, which is in very good agreement with the PDG value. Several new experiments, which pretend to remeasure α with better statistics, are still under way. So far, the latest preliminary result from the KLOE collaboration [31], $\alpha = -0.027 \pm 0.004_{\rm stat}\ {}^{+0.004}_{-0.006\rm syst}$, is based on smaller statistics, which comprises 0.65×10^6 $\eta \to 3\pi^0$ decays, and is in agreement with the PDG value within the errors.

The experimental study of the $\eta \to 3\pi^0$ decay has recently become of special interest because of new results of the NA48/2 Collaboration [32] that were obtained from the analysis of $K^+ \to \pi^+\pi^0\pi^0$ decays, where a significant cusp effect was observed in the $\pi^0\pi^0$ invariant-mass spectrum close to the $\pi^+\pi^-$ threshold. The cusp occurs because the $K^+ \to \pi^+\pi^+\pi^-$ decay contributes via the $\pi^+\pi^- \to \pi^0\pi^0$ charge exchange reaction to the $K^+ \to \pi^+\pi^0\pi^0$ decay amplitude. The cusp characteristics were used for the experimental determination of the $\pi\pi$ scattering length combination $a_0 - a_2$, the χPT prediction for which is 0.265 ± 0.004 [33]. The method for the determination of $a_0 - a_2$ from the analysis of the $\pi^0\pi^0$ invariant-mass spectrum from the $K^+ \to \pi^+\pi^0\pi^0$ decays has been presented by Cabibbo [34]. A cusp effect in the $\eta \to 3\pi^0$ decay, occurring due to the $\eta \to \pi^+\pi^-\pi^0$ decay contribution, is expected to be less significant [35]. This makes it less attractive for the experimental extraction of the $\pi\pi$ scattering lengths, but neglecting the cusp effect in the analysis of the z distribution could result in the wrong experimental value for α. In a situation like this, a new, high-statistics measurement of the $\eta \to 3\pi^0$ decays with good resolution in the $\pi^0\pi^0$ invariant mass and in the variable z is desirable.

A new precision measurement of the slope parameter α for the $\eta \to 3\pi^0$ decay based on 3×10^6 detected $\eta \to 3\pi^0$ event was made by the Crystal Ball Collaboration at MAMI-C [9]. These data are used to look for a cusp structure in the $\pi^0\pi^0$ invariant-mass spectrum and for understanding of the cusp effect on the resulting α parameter. The proposed new high statistics measurement of the $\eta \to 3\pi^0$ decay will collect about 10 times more data. It will allow a more careful investigation of the slope, in particular, establish existence of the second order term of the slope, and effect of the cusp in the $\pi\pi$ scattering on the structure of the Dalitz plot.

2.3 CP and C forbidden η decays

The CB@AGS has produced the first upper limit for the CP–forbidden $\eta \to 4\pi^0$ decay [4]:

$$BR(\eta \to 4\pi^0) < 6.9 \times 10^{-7}. \tag{5}$$

Combined with $\Gamma(\eta \to \text{all}) = 1.29 \pm 0.07\,\text{eV}$, this gives $\Gamma(\eta \to 4\pi^0) < 8.9 \times 10^{-4}$ eV. No events were found in a sample of 3×10^7 η decays produced near threshold in $\pi^- p \to \eta n$ close to threshold. To evaluate the sensitivity of this test, note that the η meson is an eigenstate of the CP operator. This allows for a comparison with a related but CP-allowed decay. The decay of a hypothetical η meson, the η_{hyp}, with $J^{PC} = 0^{++}$ into $4\pi^0$ is allowed. As η_{hyp} does not exist, we use instead $f_0(1500) \to 4\pi^0$. The f_0 has the same quantum numbers as the η except for its positive parity. The experimental value for the partial width is $\Gamma(f_0 \to 4\pi^0) = 33$ MeV. The ratio of the phase space is $\Phi(\eta \to 4\pi^0)/\Phi(f_0 \to 4\pi^0) = 4.7 \times 10^{-8}$ [36], so we might expect $\Gamma(\eta_{hyp} \to 4\pi^0) \simeq 1.6\,\text{eV}$. Thus, the CP-violating amplitude for $\eta \to 4\pi^0$ compared to a comparable, allowed decay is

$$A_{\not{cp}}/A_{cp} < \left[\frac{8.9 \times 10^{-4}\,\text{eV}}{1.6\,\text{eV}}\right]^{\frac{1}{2}} = 2.3 \times 10^{-2} \tag{6}$$

at 90% CL.

The η has the charge–conjugation eigenvalue $C = +1$, and the $\pi^0\pi^0\gamma$ system with $J^P = 0^-$ has $C = -1$. Thus, the decay $\eta \to \pi^0\pi^0\gamma$ is strictly forbidden by C invariance. This decay would be an isoscalar electromagnetic interaction of hadrons. It has been suggested that there may

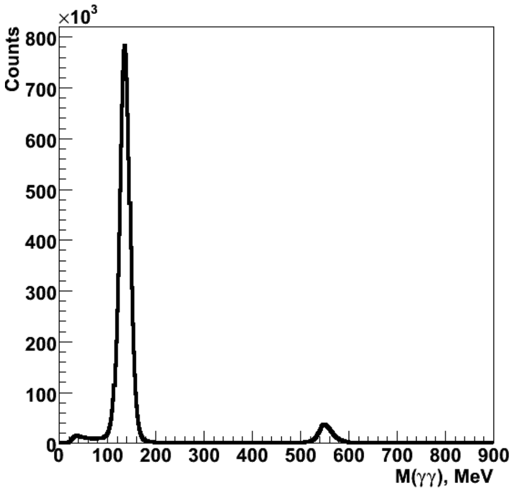

Figure 4: Experimentally measured invariant mass spectrum of two photons produced in reaction $\gamma p \to \gamma\gamma p$ with 1.505 GeV MAMI-C beam.

exist an isotensor electromagnetic interaction with a C-violating component [37, 38]. The decay $\eta \to \pi^0\pi^0\gamma$ provides an opportunity to search for such an exotic interaction; it would be a clear signal for New Physics.
The first search for $\eta \to \pi^0\pi^0\gamma$ was reported recently by the CB@AGS [5] from a sample of 3.0×10^7 η's. Candidate events in the signal region are predominantly ($\sim 85\%$) due to $\eta \to 3\pi^0$ decay with overlapping photon showers; the rest are due to $2\pi^0$ production with a split-off photon. The net yield is no events resulting in

$$BR(\eta \to \pi^0\pi^0\gamma) < 5 \times 10^{-4} \text{ at the 90\% C.L.} \tag{7}$$

This corresponds to $\Gamma(\eta \to \pi^0\pi^0\gamma) < 0.6\,\text{eV}$. To evaluate the sensitivity of our result, we can compare our upper limit of this decay rate with the measured decay rate of a suitable, C-allowed meson decay. For this purpose, we should not use the otherwise obvious decay mode $\eta \to \pi^+\pi^-\gamma$ because this decay is suppressed by the $U_A(1)$ anomaly [39]. Also, $\rho \to \pi^0\pi^0\gamma$ is not suitable because it is an isovector. The $f_0 \to \pi^0\pi^0\gamma$ decay has not been measured. For our purpose, we can use the $\rho \to \pi^+\pi^-\gamma$ decay, which has a width of 1.5 MeV. This should be adjusted for the difference in phase space [40], Clebsch-Gordan coefficients, and the angular-momentum barrier factor to account for the fact that the $2\pi^0$ system in $\eta \to \pi^0\pi^0\gamma$ decay is in a relative D-state, while the $\pi^+\pi^-$ pair in $\rho \to \pi^+\pi^-\gamma$ is mainly a P-state. The difference for the quadrupole transition involved in $\rho \to \pi^+\pi^-\gamma$ is of order $(kL)^4$, where k is the photon momentum and L is the interaction radius. We estimate that $kL \simeq \frac{1}{2}$ [41]. The decay rate for a C-allowed transition to $\pi^0\pi^0\gamma$ is thus 1.5 MeV. The sensitivity of the search for $\eta \to \pi^0\pi^0\gamma$ is

$$A^S_{\not c} / A^S_c \le \left[\frac{0.64 \text{ eV}}{1.5 \times 10^6 \text{ eV}}\right]^{1/2} = 8 \times 10^{-3} \ ,$$

where $A^S_{\not c}$ is the C-violating, isoscalar, electro-strong *amplitude*, and A^S_c is the C-allowed amplitude. This is the most sensitive limit on an isoscalar C-violating electro-strong reaction.
The radiative decay $\eta \to \pi^0\pi^0\pi^0\gamma$, is strictly forbidden by charge-conjugation invariance. There are seven photons in the final state, which explains the need for a 4π acceptance detector. Recently a first ever upper limit for the decays was also reported by the CB@AGS [5]

$$BR(\eta \to \pi^0\pi^0\pi^0\gamma) < 6 \times 10^{-5}, \tag{8}$$

This is a test of an isovector electromagnetic interaction of hadrons. To evaluate the sensitivity of this test, we proceed as follows. An allowed strong 3π meson decay is $\omega \to \pi^+\pi^-\pi^0$, which has a width of 7.6 MeV. We estimate the radiative decay to be $\alpha = 1/137$ times the corresponding

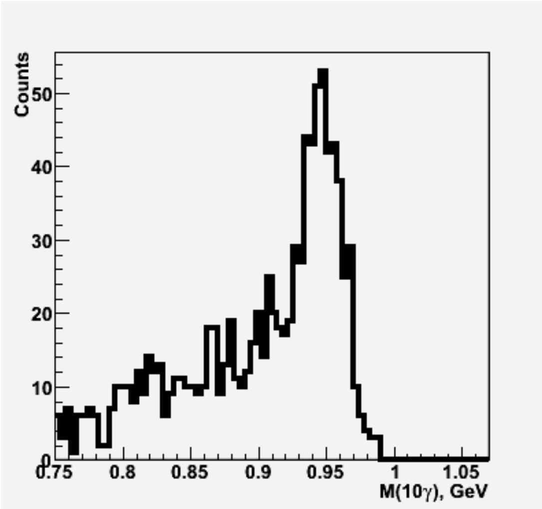

Figure 5: Experimentally measured invariant mass spectrum of ten photons produced in reaction $\gamma p \to 10\gamma p$. The peak in the figure comes from the $\eta' \to \eta\pi^0\pi^0$ decay followed by $\eta \to 3\pi^0$.

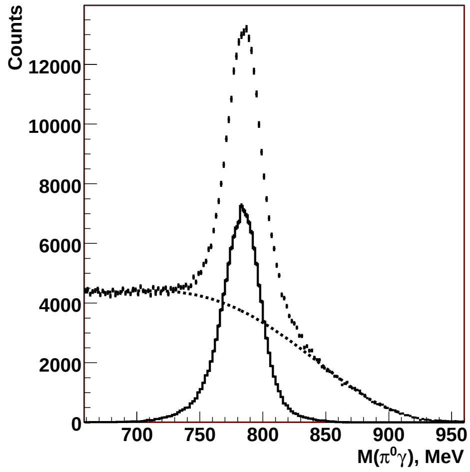

Figure 6: Invariant mass of $\pi^0\gamma$ produced in $\gamma p \to \pi^0\gamma p$. The peak is from $\omega(782) \to \pi^0\gamma$. The solid line shows the results of the Monte Carlo simulation.

hadron decay width. After adjusting for the spin-statistics and symmetry factor, the C-allowed $3\pi^0\gamma$ decay width is 6.8×10^3 eV. The sensitivity is

$$A^V_{\not{c}}\ /A^V_c\ \leq \left[\frac{7.7 \times 10^{-2}\text{eV}}{6.8 \times 10^3\ \text{eV}}\right]^{1/2} = 3 \times 10^{-3}\ ,$$

where $A^V_{\not{c}}$ is the isovector C-violating amplitude. This is the best available limit on the absence of a C-violating, isovector *amplitude.*

The decay of a neutral, flavorless, $C = +1$, pseudoscalar meson into three photons is forbidden rigorously by C-invariance. The 3γ decay should be small as it is a third order electromagnetic interaction and $\alpha^3 = 4 \times 10^{-7}$. The rate is further suppressed by substantial factors coming from phase space and angular momentum barrier considerations [41]. The decay $\eta \to 3\gamma$ can be isoscalar or isovector and even the hypothetical isotensor interaction. The Particle Data Group [26] lists the upper limit for the $\eta \to 3\gamma$ branching ratio as 5×10^{-4}.

The CB@AGS has produced a new result which is [7, 42]

$$BR(\eta \to 3\gamma) < 4.0 \times 10^{-5} \tag{9}$$

at the 90% C.L. The largest background in this experiment is from $\eta \to 3\pi^0 \to 6\gamma$ decay, $BR(\eta \to 3\pi^0) = 0.32$, when photon showers overlap in the detector. The background from $\eta \to \pi^0\gamma\gamma$ decay when two photons overlap is insignificant because of the smallness of the branching ratio, $BR(\eta \to \pi^0\gamma\gamma) = 3 \times 10^{-4}$. The background from $\eta \to 2\gamma$ with two split-offs is greatly suppressed in our analysis.

A run of about 800 hours yielding over 2.5×10^8 η's would improve the current upper limits on the branching ratio listed above by factor of 10.

3 Experimental issues

3.1 Experimental apparatus

The proposed measurement will use the existing apparatus located in the real photon beam of MAMI. The experiment uses the Glasgow–Edinburgh–Mainz photon tagger, the Crystal Ball

photon spectrometer, TAPS as a forward detector, and a particle identification detector (PID-II) which is a cylinder made of 24 scintillator strips 4 mm thick located around the liquid H_2 target, see Ref. [43] for details on the experimental setup. The experimental apparatus provides close to 4π sr coverage for outgoing photons. Protons are detected by the TAPS forward wall for $\Theta_{lab} < 21°$, and by the PID for other angles. The acceptance calculated for example for the $\eta \to \pi^0\gamma\gamma$ is about 30% when all four photons and the proton are detected.
The experimental apparatus was successfully used for our eta run in 2007 at MAMI-C. Figures 4, 5, and 6 illustrate the quality of the data obtained during the 2007 run. The MAMI experiment can detect both, neutral as well as charged decays of the eta, for example $\gamma p \to e^+e^-\gamma p$ events have been clearly detected.
In order to minimize the systematic uncertainty of the measurement we will use the simplest possible trigger requirements: the total energy in the Crystal Ball $\sim$ 360 MeV or higher, and the number of the Crystal Ball blocks fired is three or more. One CB block consists of a fixed group of 16 adjacent crystals, where at least one crystal has a deposited energy of 30 MeV or higher. The expected trigger rate is about 800 Hz at $\sim$15 nA primary beam incident on 10 μm Cu radiator and 4 mm collimator.

4 Event rates and beamtime estimate

The rate in the tagger ladder of $6 \times 10^4 \frac{e}{sec \cdot MeV}$ was used during the 2007 MAMI-C run. The two main factors that limited the beam intensity were the rate per counter in the tagger ladder, and the event rate in the two inner rings of the TAPS forward detector. In the proposed experiment we will use a 10 cm liquid hydrogen target instead of 5 cm. This will allow us to double the event rate keeping the same rate on the tagger ladder. The problem of the two inner rings of the TAPS detector was solved by replacing 18 BaF crystal in the two rings with 72 faster $PbWO_4$ (lead tungstate) crystals. The upgraded TAPS forward wall can handle higher event rates produced by the 10 cm target.

- Incoming electron beam energy: $E_0 = 1558$ MeV.
- Tagged photon energy range: $E_\gamma^t = 707 - 1450$ MeV, thus $\Delta E_\gamma = 743$ MeV.
- Electron count rate in the tagger: $N_e = 6 \times 10^4 \frac{1}{s \cdot MeV}$.
- Tagging efficiency: $\varepsilon_t \approx 70\%$.
- Tagged photon flux (average between 707 and 1450 MeV): $N_\gamma = 3 \times 10^4 \frac{\gamma}{s \cdot MeV}$.
- Number of protons in a 10 cm long LH_2 target: $N_t = 4 \times 10^{23} \frac{1}{cm^2}$.
- Eta photoproduction cross section (average between 707 and 1450 MeV): $\sigma_t(\gamma p \to \eta p) = 10\mu b$

The number of etas is

$$N_\gamma \Delta E_\gamma \Delta t N_t \sigma_t \approx 3 \times 10^5 \, \eta/h.$$

The total running time with full target of 800 hours allows us to produce about 2.5×10^8 η's. With a detection efficiency for the $\pi^0\gamma\gamma$ channel conservatively taken to be 25%, a data acquisition system livetime of 70%, and $BR(\eta \to \pi^0\gamma\gamma) = 2.2 \times 10^{-4}$, we expect about 10000 good $\eta \to \pi^0\gamma\gamma$ events. We can also can improve by an order of magnitude on the upper limits of $\eta \not\to 2\pi^0\gamma$, $\eta \not\to 3\pi^0\gamma$, $\eta \not\to 3\gamma$, and $\eta \not\to 4\pi^0$.
We estimate needing 100 hours of empty target data for background measurements, and 50 hours for trigger studies.

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